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SUSY $N = 2$ hyperelliptic curve from $N = 1$ effective potential

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Abstract

We derive the singularity conditions of the $N = 1$ generalized (general yukawa couplings and quark masses) form of hyperelliptic curves of $SU(N_c)$ with N_f flavors. The results reproduce the known form of $N = 2$ curves when the yukawa couplings and the quark masses reduce to those of $N = 2$. We obtained these curves by determining the dependence of the unbroken $SU(2)$ gaugino condensation on the couplings in the moduli source terms which break $N = 2$ SQCD to $N = 1$ $SU(N_c)$ gauge theory with the quarks and the adjoint matter, Φ . The degenerate component of the diagonalized classical vacuum expectation value of Φ is shown to be explicitly written in terms of these couplings, which enables us to determine the form of the gaugino condensation.

The recent developments of $N = 2$ SUSY gauge theory enable us to obtain exact descriptions of the low energy strong coupling region. According to them, the theories are parametrized by the vacuum expectation values of the gauge invariant operators which form the quantum moduli space. The singularities of the quantum moduli space correspond to the appearance of massless solitons. Therefore when massless solitons appear, the (hyperelliptic) curves which describe the quantum moduli space have vanishing cycles. If we add the moduli source terms which break $N = 2$ to $N = 1$, all the $N = 2$ vacua without massless solitons lift and only the $N = 2$ vacua with massless solitons remain as $N = 1$ vacua [1], [2].

On the other hand, we may start from the $N = 1$ theory with the superpotential containing the $N = 2$ tree-level superpotential and the moduli source terms which reduce $N = 2$ SUSY to $N = 1$. When we investigate the low energy strong coupling region of the theory, we can make use of the technique to decide the form of $N = 1$ effective superpotentials by symmetry. From the low energy effective superpotential determined in this way, we can derive the moduli, namely, the v.e.v.s of the gauge invariant operators which describe the quantum vacua. We can show that a hyperelliptic curve becomes singular at these v.e.v.s and the curve corresponds with that of $N = 2$ theory in the $N = 2$ limits of the couplings.

Indeed, for $SU(N_c)$ pure super Yang-Mills, Elitzur et al. [3] have obtained the $N = 1$ effective superpotential from which the $N = 2$ curve, [4], [5] is reproduced. The approach has been generalized to the other classical groups in the case of pure SYM in ref [6]. Consistency with $N = 2$ results has been further checked for $SO(5)$ with one flavor in [7] and the authors have also applied this method to G_2 SUSY gauge theory.

Our purpose in this paper is to examine the case of SQCD ($SU(N_c)$) with general flavors. The authors of [3] have also tried this case. They “integrate in” [8] the adjoint matter from the $N = 1$ low energy $SU(2)$ SQCD effective potential and get the superpotential with the adjoint matter in the high energy. This potential leads to $SU(2)$ SQCD effective potential in the low energy when the adjoint matter is integrated out around the classical vacua with $SU(2) \times U(1)^{N_c-2}$ gauge symmetry. Using the equations of motion for this potential, they reproduced the $N = 2$ hyperelliptic curve [9] in the case of one flavor SQCD. But the known $N = 2$ curves for SQCD with general flavors have not been

re-derived by this method. We shall derive the singularity conditions for the curves of SQCD from the low energy superpotential which is obtained by integrating all the matter. This low energy potential contains the $SU(2)$ gaugino condensation. The key point is that we can determine the dependence of this gaugino condensation on the couplings g_r ($r = 2, 3, \dots, N_c$) in Eq.(1) in this paper. As the result of that, we can show that the method which the authors of [3] have used for SYM successfully can also be applicable to the case of SQCD with general flavors. These curves derived by this method reproduce the curves of $N = 2$ SQCD when the couplings reduce those of $N = 2$ SQCD.

We first briefly summarize the methods of our derivation. In this paper we denote quarks by Q^r and anti-quarks by \tilde{Q}^s which are the $N = 1$ chiral superfields in the fundamental and anti-fundamental representations of $SU(N_c)$ gauge symmetry, where $r, s = 1, 2, \dots, N_f$ are flavor indices. They make up $N = 2$ hypermultiplets together. We also denote by Φ an $N = 1$ adjoint chiral superfield which is in an $N = 2$ vector multiplet.

1 By starting from the following superpotential,

$$W_{tree} = \sum_{k=2}^{N_c} \frac{g_k \text{Tr} \Phi^k}{k} + \lambda Q \Phi \tilde{Q} + m_Q Q \tilde{Q}, \quad (1)$$

we can perturb $N = 2$ SQCD into $N = 1$ theory. Here, λ, m_Q are the couplings with the flavor indices,¹ and g_k is the coupling to the gauge invariant operator, $\frac{\text{Tr} \Phi^k}{k}$. Then assuming the classical vacua, $\langle \tilde{Q} \rangle = 0, \langle Q \rangle = 0$ for all the quarks, we get the classical v.e.v of Φ , Φ_{cl} from the equations of motion. Classically, the generic unbroken nonabelian gauge symmetry is $SU(2)$ and we can determine the degenerate component M of $\Phi_{cl} = \text{diag}(M, M, M_3, M_4, \dots, M_{N_c})$, where $M \neq M_a \neq M_b, (a \neq b)$ from the equation of motion.

2 Next, we integrate out massive particles, except the quarks which transform as the fundamental and anti-fundamental representations of $SU(2)$ (we denote these quarks as the $SU(2)$ quarks below.) around the chosen vacua and add the low energy effective superpotential of $N = 1$ $SU(2)$ massless SQCD, W_d . In addition to them, we must consider the other quantum corrections, W_Δ using the notation of ref [8]. W_Δ is the potential which may generate in the low energy theory because of the quantum effects of the high energy theory. We can not exclude this term by the symmetry and the classical

¹We abbreviate flavor indices of λ and m_Q , and the summation over them in (1).

limits. We assume below that W_Δ is zero and compare the results under this assumption with the exact results. Finally, we get the low energy superpotential with only the $SU(2)$ quarks, $W_L = W_{cl} + W_d$. Here, we denote W_{cl} as the potential substituted in W_{tree} by Φ_{cl} and the quark v.e.v.s except the $SU(2)$ quarks. Here, we assume that the masses of the quarks except the $SU(2)$ quarks, that is M_a ($a = 3, 4, \dots, N_c$), are very large, so that the v.e.v.s of these quarks are zero.

3 Then we integrate out the $SU(2)$ quarks and get the low energy superpotential W_{LL} which contains the $SU(2)$ gaugino condensation. We can determine this gaugino condensation in terms of g_{N_c} , g_{N_c-1} , m_Q and the dynamical scale of the original theory, Λ . Here we have to limit the range of parameters so that the assumption in 1, $\langle \tilde{Q} \rangle = 0$, $\langle Q \rangle = 0$ is compatible with non zero v.e.v.s of the $SU(2)$ invariants which consist of the $SU(2)$ quarks. Moreover in the case of $N_f > 3$, these v.e.v.s of the $SU(2)$ invariants must be far away from some special points, where massless solitons (dual quarks) contribute. But the vacua we analyze are most generic in the allowed range, so the results become consistent with $N = 2$ theory and may be exact even in the range outside the above restriction because of holomorphy.

4 By taking the derivative of the W_{LL} with respect to g_r , we obtain the quantum moduli as in the case of SYM in [3]. In this method, they are only the semi-classical moduli, but in fact it will turn out to be exact. In this way we can relate the classical moduli to the quantum moduli.

5 We can then write the characteristic equation for Φ_{cl} in terms of the quantum moduli, which leads to the condition of vanishing cycles of $N = 2$ curve.

Now let us proceed to calculations according to the above scenario. The classical equation of motion for Φ is

$$\sum_{k=2}^{N_c} g_k \Phi^{k-1} - \frac{1}{N_c} \sum_{k=2}^{N_c} g_k \text{Tr} \Phi^{k-1} = 0. \quad (2)$$

The second term has its origin in the fact that Φ is traceless. Namely, the equation of motion for Φ needs a Lagrange multiplier to take account into this constraint. The equation (2) is the form after the Lagrange multiplier is eliminated. We define an equation

by

$$\sum_{k=2}^{N_c} g_k x^{k-1} - \frac{1}{N_c} \sum_{k=2}^{N_c} g_k \text{Tr} \Phi_{cl}^{k-1} = 0. \quad (3)$$

Below we often define $u_k = \langle \frac{\text{Tr} \Phi^k}{k} \rangle$, and $u_k^{cl} = \frac{\text{Tr} \Phi_{cl}^k}{k}$. Let us consider the most generic case in which the $N_c - 1$ components of the diagonalized classical solution of (2), $\Phi_{cl} = \text{diag}(M_1, M_2, \dots, M_{N_c})$ are different from each other, say $M_a \neq M_b$ ($a \neq b, a, b = 2, 3, 4, \dots, N_c$). Then (3) has $N_c - 1$ solutions which can be identified with $N_c - 1$ different components of the diagonalized classical solution of (2). Therefore the diagonalized solution of (2) has two components with the same value, M , that is $\Phi_{cl} = \text{diag}(M, M, M_3, \dots, M_{N_c})$, where $M \neq M_a, M_a \neq M_b, (a \neq b, a, b = 3, 4, \dots, N_c)$. The same logic as this is also applicable to other gauge groups besides $SU(N_c)$ and there is necessarily unbroken nonabelian gauge symmetry in any classical vacua [6].

In the particular case of $SU(N_c)$, fortunately, any component can be explicitly represented as minus the sum of the other $N_c - 1$ solutions because the trace of Φ_{cl} is zero. So we can see from (3) that the sum of the $N_c - 1$ different components is $-\frac{g_{N_c-1}}{g_{N_c}}$. Thus we have shown that there are always two $\frac{g_{N_c-1}}{g_{N_c}}$ s among the N_c components of the diagonalized classical solution of (2) when classically $SU(2) \times U(1)^{N_c-2}$ is unbroken. We have assumed here the v.e.v.s of quarks are zero. Below we define $\frac{g_{N_c-1}}{g_{N_c}}$ as M .

As a first example, let us consider the case of one flavor. After decomposing $\Phi = \Phi_{cl} + \delta\Phi$, we substitute this into (1),²

$$\sum_{k=1}^{N_c} \frac{g_k \text{Tr} \Phi^k}{k} = \sum_{k=1}^{N_c} \frac{g_k \text{Tr} \Phi_{cl}^k}{k} + \frac{1}{2} \sum_{k=1}^{N_c} g_k \text{Tr} (\delta\Phi^2 \Phi_{cl}^{k-2}) (k-1) + O(\delta\Phi^3). \quad (4)$$

In particular, the mass term for the fluctuation along the unbroken $SU(2)$ is [6], [10]

$$\begin{aligned} \frac{1}{2} m_{SU(2)} \text{Tr} \delta\Phi_{SU(2)}^2 &= \frac{1}{2} \sum_{k=1}^{N_c} (k-1) g_k \text{Tr} (\delta\Phi_{SU(2)}^2 \Phi_{cl}^{k-2}) = \frac{1}{2} \sum_{k=1}^{N_c} (k-1) g_k \text{Tr} (\delta\Phi_{SU(2)}^2) M^{k-2} \\ &= \frac{1}{2} g_{N_c} \left(\prod_{a=3}^{N_c} (M - M_a) \right) \text{Tr} \delta\Phi_{SU(2)}^2. \end{aligned}$$

We assume that $m_{SU(2)}$ is large compared with the dynamical scale of the original theory, Λ . The quarks except the $SU(2)$ quark have masses of order, M_a ($a = 3, 4, \dots, N_c$) and we

²In (4) we consider the potential that the Lagrange multiplier is not eliminated.

also assume $M_a \gg \Lambda$. Then we integrate out $\delta\Phi$ and these quarks, assuming that $m_{\text{SU}(2)}$ and M_a are so large that we can ignore the interaction terms of $\delta\Phi$ and all the quarks, $\lambda Q\delta\Phi\tilde{Q}$. We denote the dynamical scale of the low energy $SU(2)$ with one flavor, Λ_L by the relation, $\Lambda_L^5 = \Lambda^{2N_c-1} g_{N_c}^2$ [6] [10]. Finally we obtain the superpotential with only the $SU(2)$ quark,

$$W_L = (\lambda M + m_Q) Q^\alpha \tilde{Q}_\alpha + \frac{\Lambda_L^5}{Q^\alpha \tilde{Q}_\alpha} + \sum_{k=2}^{N_c} \frac{g_k \text{Tr} \Phi_{cl}^k}{k} + W_\Delta, \quad (5)$$

where $\alpha = 1, 2$ mean the $SU(2)$ indices. The first term originates by substituting Φ_{cl} into W_{tree} and the second is the $SU(2)$ dynamically generated term. In this step, we assume W_Δ is zero under the above assumption, $M_a, m_{\text{SU}(2)} \gg \Lambda$ as in [3]. Then, we integrate the $SU(2)$ quark, and we obtain the effective potential with the $SU(2)$ gaugino condensation,

$$W_{LL} = \sum_{k=2}^{N_c} \frac{g_k \text{Tr} \Phi_{cl}^k}{k} \pm 2\Lambda_L^{5/2} \sqrt{\lambda M + m_Q} = \sum_{k=2}^{N_c} \frac{g_k \text{Tr} \Phi_{cl}^k}{k} \pm 2g_{N_c} \Lambda^{N_c-\frac{1}{2}} \sqrt{\lambda M + m_Q}. \quad (6)$$

When we integrated out the fluctuation of the adjoint superfield, we assumed the v.e.v.s of the quarks to be zero. The v.e.v.s of the $SU(2)$ invariant operator, $\langle Q\tilde{Q} \rangle$ is order of $\Lambda_L^{5/2} [\lambda M + m_Q]^{-\frac{1}{2}} = \Lambda^{N_c-\frac{1}{2}} g_{N_c} [\lambda M + m_Q]^{-\frac{1}{2}}$, so we need $m_{\text{SU}(2)}^2 \gg \Lambda^{N_c-\frac{1}{2}} g_{N_c} [\lambda M + m_Q]^{-\frac{1}{2}}$. It is required that the v.e.v.s of the $SU(2)$ invariant operators should satisfy the condition like this, also in the case of SQCD with general flavors. From W_{LL} we can obtain the equations which relate the quantum moduli to the classical moduli by taking the derivative of W_{LL} with respect to g_r ,

$$u_r = u_{cl\,r} \pm 2\Lambda^{N_c-\frac{1}{2}} \left(\frac{g_{N_c} \lambda \frac{\partial M}{\partial g_r}}{2\sqrt{\lambda M + m_Q}} + \sqrt{\lambda M + m_Q} \delta_{N_c,r} \right). \quad (7)$$

From the definition of M , that is $M = \frac{g_{N_c-1}}{g_{N_c}}$, we get

$$u_r = u_{cl\,r} \pm 2\Lambda^{N_c-\frac{1}{2}} \left[\delta_{N_c,r} (\sqrt{\lambda M + m_Q} - \frac{\lambda M}{2\sqrt{\lambda M + m_Q}}) + \delta_{N_c-1,r} \frac{\lambda}{2\sqrt{\lambda M + m_Q}} \right]. \quad (8)$$

Now using the characteristic equation and the Newton formula,

$$P(x : u) = \langle \det(x - \Phi) \rangle = \sum_{k=0}^{N_c} s_k x^{N_c-k}, \quad k s_k = - \sum_{j=1}^k j s_{k-j} u_j, \quad (9)$$

we get the relations between classical s_r^{cl} and quantum s_r ,

$$\begin{aligned} s_k &= s_k^{cl} \quad k = 0, 1, 2 \cdots N_c - 2, \\ s_{N_c-1} &= s_{N_c-1}^{cl} \mp \Lambda^{N_c-\frac{1}{2}} \frac{\lambda}{\sqrt{\lambda M + m_Q}}, \\ s_{N_c} &= s_{N_c}^{cl} \mp \Lambda^{N_c-\frac{1}{2}} \frac{\lambda M + 2m_Q}{\sqrt{\lambda M + m_Q}}. \end{aligned} \quad (10)$$

Using (10) to rewrite $P(x : u_{cl}) = \sum_{k=0}^{N_c} s_k^{cl} x^{N_c-k}$ by s_k , we get

$$P(x : u_{cl})_{\pm} = P(x : u) \pm \Lambda^{N_c-\frac{1}{2}} (\lambda M + m_Q)^{-\frac{1}{2}} (\lambda x + \lambda M + 2m_Q). \quad (11)$$

and consider two vacua together by defining

$$\tilde{P}(x) \equiv P(x : u_{cl})_+ P(x : u_{cl})_- = P(x : u)^2 - \Lambda^{2N_c-1} (\lambda M + m_Q)^{-1} (\lambda x + \lambda M + 2m_Q)^2. \quad (12)$$

Either $P(x : u_{cl})_+$ or $P(x : u_{cl})_-$ has the double root at $x = M$, so we have

$$\begin{aligned} \tilde{P}(x = M) &= P(M : u_{cl})_+ P(M : u_{cl})_- \\ &= P(M : u)^2 - \Lambda^{2N_c-1} (\lambda M + m_Q)^{-1} (\lambda M + \lambda M + 2m_Q)^2 \\ &= P(M : u)^2 - 4\Lambda^{2N_c-1} (\lambda M + m_Q) = 0, \\ \left. \frac{d\tilde{P}(x)}{dx} \right|_{x=M} &= 2P(M : u) \left. \frac{dP(x : u)}{dx} \right|_{x=M} - 4\Lambda^{2N_c-1} \lambda = 0, \end{aligned}$$

which show that

$$y^2 = P(x : u)^2 - 4\Lambda^{2N_c-1} (\lambda x + m_Q) \quad (13)$$

is singular at $x = M$. This is the known $N = 2$ curve for $SU(N_c)$ with one flavor [9] when the quark masses and the yukawa couplings reduce to those of $N = 2$, that is $\lambda_{ij} = \delta_{ij}$ and $[m, m^\dagger] = 0$. The authors of [3] have derived this result by the “integrate-in” method [8] for Φ to get the superpotential with Q , \tilde{Q} , and Φ , and by the equations of motion for this potential.

Next consider the case of two flavors. We have to redefine the quarks in order to make clear the transformation property under the flavor symmetry, enhanced by the residual $SU(2)$ gauge symmetry as $Q^{2r-1,\alpha} \equiv Q^{r,\alpha}$, $Q^{2s,\alpha} \equiv \tilde{Q}^s_{\beta} \epsilon^{\alpha\beta}$ ³ and we denote

³ $\epsilon^{\alpha\beta}$ and $\epsilon_{\alpha\beta}$ are antisymmetric tensors, $\epsilon^{12} = -\epsilon^{21} = 1$ and $\epsilon_{21} = -\epsilon_{12} = 1$.

$r, s = 1, 2$ as the original flavor indices and $i, j = 1, 2, 3, 4$ as those of the enhanced flavor symmetry. α, β mean the indices of $SU(2)$ gauge symmetry. Then the $SU(2)$ gauge invariant operators are $V^{ij} \equiv \epsilon_{\alpha\beta} Q^{i\alpha} Q^{j\beta}$. As in the case of one flavor, we obtain the low energy superpotential with the $SU(2)$ quarks,

$$W_L = \sum_{s,r=1,2} (\lambda_{r,s} M + m_{Q_{r,s}}) V^{2r-1,2s} + (\text{Pf}V - \Lambda_L^4) Y + \sum_{k=2}^{N_c} g_r u_r^{cl}, \quad (14)$$

where Λ_L is the dynamical scale of $SU(2)$ with two flavors and $\Lambda_L^4 = g_{N_c}^2 \Lambda^{2N_c-2}$ [6], [10]. $\text{Pf}V$ is the Pfaffian, $\text{Pf}V = \frac{1}{2^{N_c} N_c!} \epsilon_{i_1 i_2 j_1 j_2 \dots} V^{i_1 i_2} V^{j_1 j_2} \dots$ and Y is a Lagrange multiplier superfield to the constraint for V due to the quantum effect in $SU(2)$ gauge theory with two flavors, $\text{Pf}V = \Lambda_L^4$ [11]. Integrating out V , we get the superpotential containing the $SU(2)$ gaugino condensation,

$$W_{LL} = \pm 2 \Lambda_L^2 [\det(\lambda M + m_Q)]^{\frac{1}{2}} + \sum_{k=2}^{N_c} g_r u_r^{cl} = \pm 2 \Lambda^{N_c-1} g_{N_c} [\det(\lambda M + m_Q)]^{\frac{1}{2}} + \sum_{k=2}^{N_c} g_r u_r^{cl}, \quad (15)$$

where \det means the determinant for the original flavor indices, that is the determinant of 2×2 matrix. From this we can derive the relation between the quantum moduli and the classical moduli,

$$u_r = u_r^{cl} \pm \Lambda^{N_c-1} [\det(\lambda M + m_Q)]^{\frac{1}{2}} \left[2\delta_{r,N_c} + \frac{g_{N_c}}{\det(\lambda M + m_Q)} \frac{\partial M}{\partial g_r} \frac{\partial \det(\lambda M + m_Q)}{\partial M} \right], \quad (16)$$

$$\frac{\partial M}{\partial g_r} = -\frac{M}{g_{N_c}} \delta_{r,N_c} + \frac{1}{g_{N_c}} \delta_{r,N_c-1}. \quad (17)$$

As in (10), from (16), (17) and the Newton formula,

$$\begin{aligned} s_k &= s_k^{cl} \quad k = 0, 1, 2 \dots N_c - 2, \\ s_{N_c-1} &= s_{N_c-1}^{cl} \mp \Lambda^{N_c-1} [\det(\lambda M + m_Q)]^{-\frac{1}{2}} \frac{\partial \det(\lambda M + m_Q)}{\partial M}, \\ s_{N_c} &= s_{N_c}^{cl} \mp \Lambda^{N_c-1} \left(2[\det(\lambda M + m_Q)]^{\frac{1}{2}} - \frac{M}{[\det(\lambda M + m_Q)]^{\frac{1}{2}}} \times \frac{\partial \det(\lambda M + m_Q)}{\partial M} \right) \end{aligned} \quad (18)$$

are obtained. By using these results we define $\tilde{P}(x)$ as in (12),

$$\begin{aligned} \tilde{P}(x) &= P(x : u_{cl})_+ P(x : u_{cl})_- \\ &= P(x : u)^2 - \Lambda^{2N_c-2} \left(x [\det(\lambda M + m_Q)]^{-\frac{1}{2}} \frac{\partial \det(\lambda M + m_Q)}{\partial M} \right. \\ &\quad \left. + 2[\det(\lambda M + m_Q)]^{\frac{1}{2}} - \frac{M}{[\det(\lambda M + m_Q)]^{\frac{1}{2}}} \frac{\partial \det(\lambda M + m_Q)}{\partial M} \right)^2. \end{aligned} \quad (19)$$

Substituting $x = M$, and by the condition of the double root we get

$$\tilde{P}(x = M) = P(M : u^{cl})_+ P(M : u^{cl})_- = P(M : u)^2 - 4\Lambda^{2N_c-2} \det(\lambda M + m_Q) = 0, \quad (20)$$

$$\left. \frac{d\tilde{P}(x)}{dx} \right|_{x=M} = 2P(M : u) \left. \frac{dP(x : u)}{dx} \right|_{x=M} - 4\Lambda^{2N_c-2} \frac{\partial \det(\lambda M + m_Q)}{\partial M} = 0. \quad (21)$$

The above equations mean that the curve described by

$$y^2 = P(x : u)^2 - 4\Lambda^{2N_c-2} \det(\lambda x + m_Q) \quad (22)$$

is singular at $x = M$. This is the $N = 2$ curve for $SU(N_c)$ with two flavors [9] in the $N = 2$ limits of the couplings. We have neglected W_Δ in the steps above as before.

We now consider the general theory with N_f flavors. ($2 < N_f < N_c$)

$$W_L = \sum_{s,r=1}^{N_f} (\lambda_{r,s} M + m_{Q_{r,s}}) V^{2r-1,2s} + (\text{Pf}V)^{\frac{1}{N_f-2}} (2 - N_f) \Lambda_L^{\frac{N_f-6}{N_f-2}} + \sum_{k=2}^{N_c} g_r u_r^{cl}, \quad (23)$$

where we redefine the quarks as before, $Q^{2r-1,\alpha} \equiv Q^{r,\alpha}$, $Q^{2r,\alpha} \equiv \tilde{Q}^r_\beta \epsilon^{\alpha\beta}$, $V^{ij} \equiv \epsilon_{\alpha\beta} Q^{i\alpha} Q^{j\beta}$, and we denote the relation between the original dynamical scale, Λ and that of the low energy $SU(2)$ with N_f flavors, Λ_L as $\Lambda_L^{6-N_f} = g_{N_c}^2 \Lambda^{2N_c-N_f}$ [6], [10]. Here, the second term in W_L is the effective superpotential determined by the symmetry in $SU(2)$ gauge theory with N_f flavors, [11] and $r, s = 1, 2 \cdots N_f$ are the indices of the original flavor symmetry while $i, j = 1, 2, 3 \cdots 2N_f$ are the indices of the flavor symmetry, enhanced by the residual $SU(2)$ gauge symmetry. From the equations of motion for V , we get $\langle \text{Pf}V \rangle = g_{N_c}^{N_f} [\det(\lambda M + m_Q)]^{\frac{N_f-2}{2}} \Lambda^{N_f(N_c-N_f/2)}$ and obtain the superpotential with the $SU(2)$ gaugino condensation,

$$W_{LL} = \sum_{k=2}^{N_c} g_r u_r^{cl} \pm 2[\det(\lambda M + m_Q)]^{\frac{1}{2}} g_{N_c} \Lambda^{N_c-N_f/2}, \quad (24)$$

where \det means the determinant for the $N_f \times N_f$ matrix of the original flavor symmetry. We assume that W_Δ is zero. It is important to note that in the case of $N_f > 3$, the couplings must be far away from the points where $\langle \text{Pf}V \rangle = 0$ is satisfied, because the dual quarks contribute there. Of course, in addition to this condition, $\langle V \rangle$ must be much smaller than $m_{SU(2)}^2$ as in the case of one flavor.

(24) is the same as (15) if we replace the determinant in (15) by that of the $N_f \times N_f$ flavor matrix and replace Λ^{N_c-1} in (15) by $\Lambda^{N_c-N_f/2}$. So we can conclude from (24) that the curve,

$$y^2 = P(x : u)^2 - 4\Lambda^{2N_c-N_f} \det(\lambda x + m_Q) \quad (25)$$

becomes singular at $x = M$, in the same way as (16), (17), (18), (19), (20) and (21) in the case of two flavors. This curve corresponds to the $N = 2$ curve for $SU(N_c)$ N_f flavor SQCD [9] in the $N = 2$ limits of the couplings.

If we scale $\tilde{\Lambda}^{2N_f} = \det(m_Q)\Lambda^{2N_c-N_f}$ with large m_Q , then in (24) we get the same superpotential of [3] by which the authors have derived the $N = 2$ SYM curve [4], [5]. That is, it corresponds to integrating all the quarks first and considering remaining $N = 2$ pure gauge theory by perturbing with W_{tree} without quark mass terms and yukawa terms.

We have considered the case of $N_f < N_c$ until now. For $N_f \geq N_c$, if we add the baryon source terms,

$$b_{i_1 i_2 \dots i_{N_c}} B^{i_1 i_2 \dots i_{N_c}} + \tilde{b}^{i_1 i_2 \dots i_{N_c}} \tilde{B}_{i_1 i_2 \dots i_{N_c}} \quad (26)$$

to W_{tree} , W_{LL} is the same as in $N_f < N_c$ in our method because these terms do not seem to contribute when M_a are very large. Then the curves are the same as in $N_f < N_c$ and correspond with the results of [9] in the case of $N_f < 2N_c - 1$. For $N_f = 2N_c - 1, 2N_c$, we need more modifications of this method to be consistent with the $N = 2$ results of [9]. Compared with [12], our results are the same as in $N_f < N_c$. But in $N_f \geq N_c$, their results are different from ours. It may be due to the constant shifts of the renormalization for the moduli because the moduli are composite.

In summary, we have derived the conditions for the singularities of the curves with general yukawa couplings and quark masses in the case of $SU(N_c)$ with flavors by using the $N = 1$ effective superpotential with the gaugino condensation. In the $N = 2$ limits of the couplings, these curves are the same as the known ones of $N = 2$ SQCD. These results are consistent with condensation of massless solitons that we expect from $N = 2$ theory.

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